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INTERDEPENDENCE BETWEEN DYNAMIC SURGE MOTIONS OF PLATFORM AND TETHERS FOR A DEEP WATER TLP

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SUMMARY

The tethers of tension leg platforms (TLP's) undergoing surge motions are subjected to inertia and hydrodynamic loads. The purpose of this paper is to present an investigation into the effects of the tether curvature caused by these loads. The investigation is conducted by solving the coupled equations of surge motion of the tethers/platform system. The solution method presented here involves no linearization of the equations of motion. For a typical TLP in 600 m deep water it is found that the effect of the tether curvature on the TLP surge motion is not significant. However, the amplitude of the surge restoring force supplied by the curved tethers is considerably larger (by a factor of about three) than would be the case if the tethers were straight. To ensure the reliable performance of deep water TLP's, tether/platform connections must be provided with sufficient horizontal load capacity to accommodate this increased force.

1. INTRODUCTION

At the request of the Minerals Management Service, U.S. Department of the Interior, the National Bureau of Standards is engaged in an effort aimed at providing an independent evaluation of factors affecting the reliability of tension leg platforms (TLP's). One topic of interest in this context is the effect on platform behavior of the tether curvature induced by platform surge motions. The purpose of this paper is to present an investigation into this topic.

A preliminary phase of this investigation was devoted to examining the extent to which the tether curvature depends upon the frequency of typical surge motions occurring in extreme storms [1]. To reduce the amount of computation, a simplified model was used for this purpose, in which it was assumed that the TLP surge motions are harmonic and have prescribed frequencies and amplitudes consistent with typical effects of extreme storms. For a TLP in 600 m deep water, it was found that for TLP motions due to wind excitation or secondary wave effects--which have dominant frequencies of the same order as, or lower than, the nominal natural frequencies of the platform--the tether curvature is negligible. However, for motions due to direct wave effects--the frequencies of which are higher than the natural freqency of the TLP by a factor of about five or larger--the tether curvature induced by the harmonic TLP surge motion was found to be sufficient to cause: (1) a threefold increase of the restoring force (i.e., of the horizontal projection of the tether tension at the heel of the platform), and (2) a significant time lag between the harmonic function fitted to the restoring force on the one hand, and the prescribed harmonic TLP motion on the other [1].

The question arises whether these effects would in turn cause the TLP surge motion to differ significantly from the motion calculated by assuming the tethers to be straight. For typical TLP's the restoring force--even if increased to account for tether curvature--is small compared to the inertial and the environmental force. It would therefore appear that the effects of tether curvature on the TLP motion are also small. However, a statement to this effect cannot be made on the basis of the approach just described, since in this approach the TLP motion is prescribed *a priori* and therefore cannot be calculated as a function of the restoring force. To reach a firm conclusion concerning the interaction between the TLP surge motion and the tether curvature it is necessary to solve the *coupled* equations of surge motion of the tethers and of the TLP.

The subsequent sections of the paper present: the equations of surge motion of the tethers and of the platform; a brief discussion of the numerical method employed for their solution; results obtained for a TLP in 600 m deep water; and practical conclusions based on these results. Since for low frequecy motions the tether curvature is negligible, the coupled equations of surge motion are solved only for motions due to direct wave effects.

2. EQUATIONS OF SURGE MOTION AND THEIR SOLUTION

2.1 Equations of motion

The equations of surge motion of a tether and of the platform may be written, respectively, as:

$$(m_o + m_a) \frac{\partial^2 y(z,t)}{\partial t^2} + \lambda \left| \frac{\partial y(z,t)}{\partial t} \right| \frac{\partial y(z,t)}{\partial t} = T \frac{\partial^2 y(z,t)}{\partial z^2} \quad (1)$$

$$(M + A) \frac{d^2 y(\ell,t)}{dt^2} + n\, T \left. \frac{\partial y(z,t)}{\partial z} \right|_{z = \ell} = F_H(t) \quad (2)$$

in which m_o = mass of tether per unit length; m_a = added mass of tether; T = tether tension; n = total number of platform tethers; y = horizontal displacement; z = vertical coordinate; t = time; $m_a = 0.785\rho_w C_{a,t} D^2$; $D = 0.5\rho_w C_{d,t} D$; ρ_w = water density; D = tether diameter; $C_{a,t}$ = tether added mass coefficient; $C_{d,t}$ = tether drag coefficient associated with fluid viscosity effects; M = TLP mass; A = TLP surge added mass; ℓ =length of tethers; $F_H(t)$ = hydrodynamic force. Downdraw effects occurring during the platform motion--which can be shown to be of the order of 10 percent or less (2,3)--are not taken into account.

2.2 Assumed characteristics of platform

A schematic representation of the TLP is shown in fig. 1 [2,3]. It is assumed that: $M = 4.5 \times 10^7$ kg; the platform is attached to the sea floor by 16 perfectly flexible tethers with length ℓ = 600m, mass m_o = 300 kg/m, and diameter D = 0.48m; and $T = 10^7$ N.

2.3 Ocean environment and hydrodynamic forces

The added mass and drag coefficients for the tethers depend upon the Reynolds and Keulegan-Carpenter numbers and therefore vary along the tethers. However, their overall effect may be assumed to be equivalent, approximately, to that of the constant values $C_{a,t}$ = 0.5 and $C_{d,t}$ = 1.25 [1,4].

The hydrodynamic force $F_H(t)$ was assumed to be caused by a current with speed varying from 1.4 m/s at the surface to 0.9 m/s at the keel and having negligible speed below

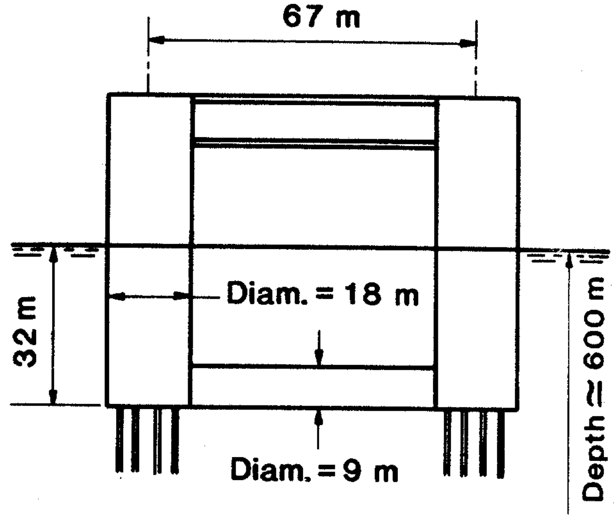

Figure 1. Platform dimensions

the keel, and by waves with period T_w = 15s and height (double amplitude) H = 25 m. The expression of F_H was assumed to be given by the Morison equation, i.e.,

$$F_H = 0.5\, \rho_w \sum_i \sum_j C_{d_{ij}} A_{p_{ij}} \left| \bar{v}_i + v_{ij} - \dot{y} \right| [\bar{v}_i + v_{ij} - \dot{y}] + \rho_w \sum_i \sum_j \Psi_{ij}\, C_{m_{ij}} \left\{ \frac{\partial v_{ij}}{\partial t} + [\bar{v}_i + v_{ij} - \dot{y}] \frac{\partial v_i}{\partial Y} \right\} \quad (3)$$

where Ψ_{ij} = elemental volume of submerged structure, $C_{m_{ij}}$ = surge inertia coefficient corresponding to Ψ_{ij}, $A_{p_{ij}}$= area of elemental volume Ψ_{ij} projected on a plane normal to the direction of the current, $C_{d_{ij}}$ = drag coefficient corresponding th $A_{p_{ij}}$, Y = horizontal distance from some arbitrary origin to center of Ψ_{ij} along direction parallel to surge motion, $\bar{v}_i$ and v_{ij} = current velocity and horizontal particle motion due to wave motion, respectively, at the center of Ψ_{ij}. It was further assumed that

$$v_{ij} = \frac{\pi H}{T_w} e^{-k_w z_i} \cos\left(k_w Y_j - \frac{2\pi}{T_w} t\right) \quad (4)$$

where the wave number is given by

$$k_w = \frac{1}{g}\left(\frac{2\pi}{T_w}\right)^2 \quad (5)$$

(3,4), and that $C_{d_{ij}} \equiv 0.6$ and $C_{m_{ij}} \equiv 1.8$ (3,4). The latter assumption yielded a calculated surge added mass of the platform $A = 3.6 \times 10^7$ kg.

2.4 Boundary conditions, and method of solution of equations of motion

The boundary condition for equation (1) at $z = 0$ is

$$y(0,t) = 0, \quad t > 0, \quad (6)$$

At $z = \ell$, the boundary condition for equation (1) is the value of $y(\ell,t)$, and it must be obtained by solving the ordinary differential equation in equation (2). In fact, the coupled system, equations (1) and (2), is an initial/boundary value problem for a nonlinear wave equation, equation (1), in which one of the boundary conditions is not given explicitly, but depends nonlinearly on the wave equation solution itself. However, as will be seen below, by organizing the calculation as a step by step marching procedure in time, with a sufficiently small time step Δt, the coupling term in equation (2) can be accurately accommodated. Computations were pursued for a total distance in time equal to 270 seconds using 1000 time steps $t_m = m\Delta t$, $m = 1, 2, \dots 1000$. Thus,

$$0 < t_1 < t_2 < t_3 < \dots < t_{1000} = 270 \text{ s} \quad (7)$$

Starting with the initial values of the cable displacement and velocities,

$$y(z,0) = 0, \quad \frac{\partial y}{\partial t}(z,0) = 0, \quad 0 \le z \le \ell, \quad (8)$$

and the compatible initial values for equation (2),

$$y(\ell, 0) = 0, \ \dot{y}(\ell, 0) = 0 \quad (9)$$

the wave equation (1) is solved numerically on $[0, t_1]$, i.e., for a time interval of .27 seconds. During this whole period, the value of the coupling term in equation (2) is set equal to its value at $t = 0$, namely zero. The numerical method for solving the wave equation is an adaptive procedure which subdivides the time interval $[0, t_1]$ into further time increments so as to accomplish the calculation to within a preset error tolerance. During that computation, the wave equation subroutine asks for boundary values, $y(\ell,t)$, at various instants on the time interval $[0, t_1]$. These values are provided by another subroutine which solves the effectively uncoupled ordinary differential equation (2), and returns $y(\ell,t)$ and $\dot{y}(\ell,t)$. When the time level $t = t_1$ is reached, the mesh values of the cable displacement are collected and interpolated by a cubic spline in the z-variable. This spline function constitutes the cable profile at $t = t_1$. By differentiating the spline and evaluating the derivative at $z = \ell$ we obtain an accurate estimate of the value of the coupling term at $t = t_1$. This new value is now inserted in equation (2) and remains constant for the entire interval $[t_1, t_2]$. In addition, the previously computed values $\dot{y}(\ell, t_1)$ and $y(\ell, t_1)$ provide new initial values for the ordinary differential equation (2). The above process is now repeated on the interval $[t_1, t_2]$ and on subsequent intervals. The basic partial differential equation solver, MOL1D [5] (also used in ref. 1) was found useful in constructing the above algorithm. This package offers a wide choice of space and time discretizations.

3. NUMERICAL RESULTS

The solution of equations 1 and 2 yielded the time history of the TLP surge motion shown in figure 2a. Figure 2b shows the solution of equation 2 in which the term $\partial y/\partial z|_{z=\ell}$ was replaced by the ratio y/ℓ, i.e., the time history of $y(\ell,t)$ obtained by assuming that the tethers are straight at all times. It can be seen that:

(1) the steady state surge motion is underestimated marginally (by about 5 percent) if the motion-induced curvature of the tethers is neglected. However, it may be expected that such an underestimation would be more significant in deeper water (say 1,000 m or 2,000 m). Recent work reported in ref. 6 suggests that this is indeed the case.

(2) The surge motion calculated by assuming the tethers to be straight overshoots before reaching a steady state. (This may be due to the zero initial conditions assumed in the calculations.) On the other hand, no overshooting occurs in the motion calculated by taking tether curvature into account, even though the same initial conditions are assumed.

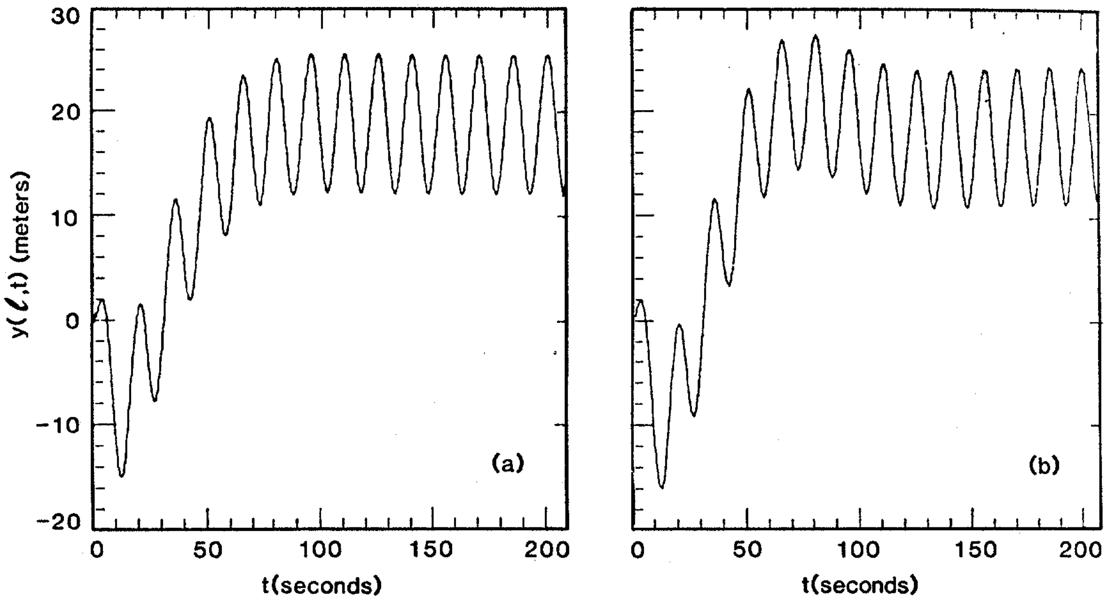

Fig. 2 - Platform surge motion: (a) curved tethers; (b) straight tethers.

Figure 4 shows the tether shapes corresponding to various positions of the platform. It is seen that for $y(\ell,t)/\ell \approx 0.042$ (i.e., near a peak, see figure 3a), the tether shape and hence the restoring force are almost the same as in the case of a straight tether. However, for $y(\ell,t)/\ell \approx 0.03$ there are substantial differences; the restoring force is almost zero for the curved tether, rather than about 0.03T, as would be the case if the tethers were straight.

It is noted that the amplification of the restoring force would have been somewhat different had wind loads and random wave loads been taken into account in the modeling of the surge motion. Also, the results of the computations depend upon the assumptions concerning the hydrodynamic loads, particularly those acting on the tethers. Associated with these assumptions are inevitable uncertainties and approximations. Nevertheless, the results presented in this paper remain valid at least in a qualitative sense.

The time history of the nondimensionalized TLP surge motion y/ℓ calculated by taking tether curvature into account is also represented in figure 3a for comparison with the time history of the time derivative $\partial y/\partial z|z=\ell$, which is shown in figure 3b. It can be seen that: (1) the derivative $\partial y/\partial z|z=\ell$ (and, therefore, the restoring force) lags the nondimensionalized horizontal displacement of the platform y/ℓ by approximately one fifth of a period, and (2) the derivative $\partial y/\partial z|z=\ell$ has an amplitude about three times larger than that of the ratio y/ℓ.

It can be concluded from figures 2 and 3 that the amplitude of the restoring force is about three times larger than would be the case if the tethers were assumed to be straight, a result similar to that obtained in ref. 1. Interestingly, in spite of this increase, the mean surge motion (though not the fluctuating part) is larger than in the case of the straight tethers (figure 2). The writers tentatively ascribe this apparent anomaly to the time lag between restoring force and surge motion noted earlier.

4. CONCLUSIONS

From the results presented in the preceding sections, it is concluded that, for the platform considered in this paper:

1) the tether curvature induced by the platform surge motion does not affect that motion significantly

2) the amplitude of the restoring force supplied by the curved tethers is considerably larger than would be the case if

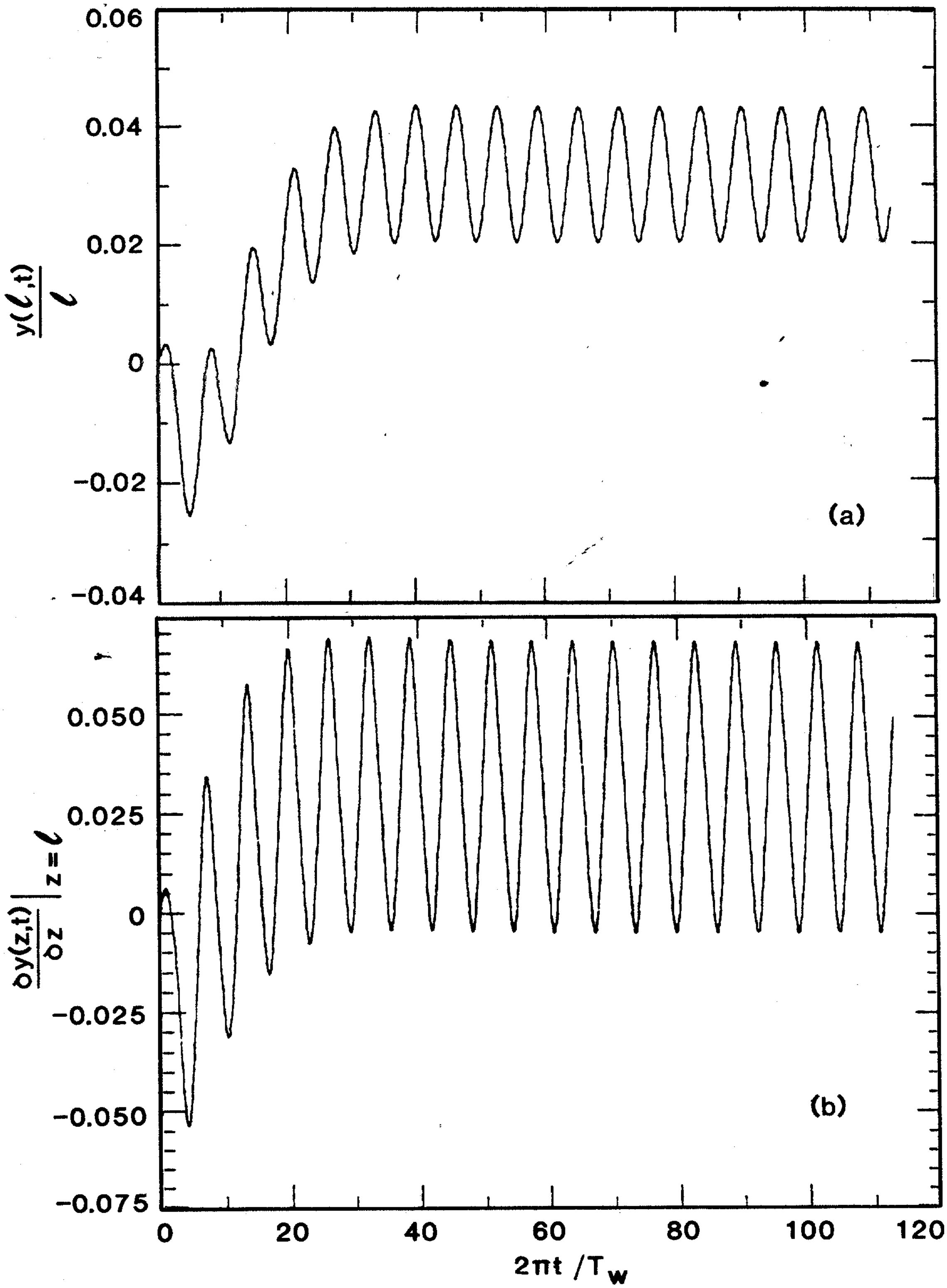

Fig. 3 - (a) Ratio $y(\ell, t)/\ell$; (b) Slope $\partial y(z, t)/\partial z|_{z = \ell}$.

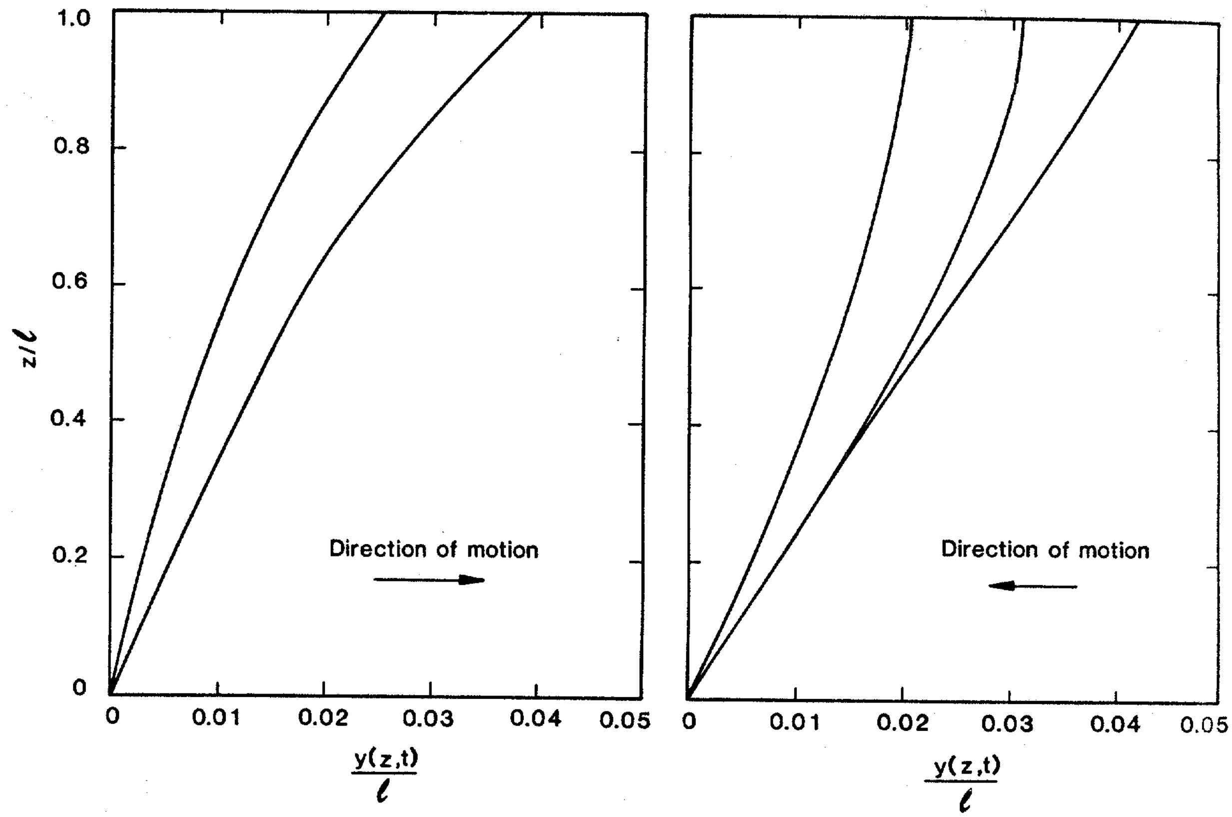

Fig. 4 - Tether shapes for various TLP positions.

the tethers were straight, by a factor that depends upon the actual hydrodynamic loads and was estimated in this paper to be about three.

From the second conclusion it follows that to ensure the reliable performance of deep water TLP's, tether/platform connections must be provided with sufficient horizontal load capacity to accommodate the increased restoring force supplied by the curved tethers.

5. ACKNOWLEDGEMENTS

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